Appendix 1: The Labour Involved in Constructing the Monument

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Calculations of the labour involved in constructing the monument are based on Startin (1982) for the excavation of the ditches, Hornsey (1987) for the excavation and erection of the stones and Startin (1983) for the transportation of the stones.

THE DITCHES

Startin's conclusion that a prehistoric team of a picker, shoveller and an appropriate number of basket carriers could excavate 24 cu ft of gravel per hour, that is 0.68 m^3, was used as the basis for estimate of labour required. There are seven approximately straight sections to the northern ditch. Assuming the same number for the southern ditch, then 14 sections of the ditches could be dug simultaneously. The volume to be excavated was 3435.32 m^3 or 245.38 m^3 per section. It is possible for three teams of picker and shoveller to dig each section, as the face has an average width of 5.84 m, so each team would have to excavate 82.79 m^3 which equates to approximately 120 hours for 84 people digging. Assuming basket chains were used, then at one person each 2 m, ten people would be in a chain and could serve two sections of ditch, ie six digging teams. So 70 people are required for removing spoil. Therefore it appears that a minimum of 154 people could excavate the ditches in 15 days.

THE STONES

Estimates of the sizes of the stones were based on measurements of stones from The Rollright Stones (Lambrick 1988), Rhos-y-Clegyrn (Lewis 1974) and standing stones at The Devil's Quoits excavated by Grimes (1943–4). It appears from these measurements that it is reasonable to estimate that about 15% of the length of a stone was underground and that the average width of a stone approximated to the width of the stonehole. If a stonehole had one long side, which was vertical, then the thickness of the stone has been estimated as half the width of the stonehole, otherwise the thickness of the stone was estimated as one-third the width of the stonehole.

The masses of the stones were calculated from their estimated volumes, derived from the above estimates, and the density of the conglomerate of which they were composed. The density of 1400 kg^{-3} was calculated from an experiment with a piece of conglomerate from Gravelly Guy.

Newtonian mechanics was used to calculate the minimum number of people and ropes and the minimum radius of levers to raise the stones from an angle of 24° to the horizontal. This initial angle is achieved by preparing the stonehole with one sloping side, and possibly a bank, and letting the stone slide into the hole under the effect of gravity. Newtonian mechanics was also used to calculate the minimum number of people and the minimum radius of levers to lift one end of a stone during extraction. For example, the heaviest stone would require 261 people without levers and 55 people with 5:1 levers of radius 0.10 m to be erected, and the lightest stones would require three people without levers and one person with a 5:1 lever of radius 0.05 m to be erected. Allowing a day for the digging of the stoneholes (the largest stonehole could be dug by one digging team in five hours), and about two hours for erection of a stone, then 60 people (five to prop and pack the stone) could erect all the stones in about 18 hours. So overall a minimum of three days would be needed to erect all the stones.

To calculate the number of people to transport the stones Newtonian mechanics was used, together with an estimate of the coefficient of friction for grass and wood from Atkinson's experiment (1960) on the downs by Stonehenge. The heaviest stone would require at least 36 people to transport it and would take approximately ten minutes to move it 300 m.

To calculate number of people to erect a stone (Table 22)

Taking moments about the base of the stone (Fig. 58) we have

$$F_i \sin\theta \times \frac{3}{4} \times l = \frac{1}{2} \times lmg\cos\theta$$

where F_i is the force applied to the stone horizontally, l is the length of the stone, m its mass and g the acceleration due to gravity. Rearranging the equation gives

$$F_i = \frac{2 \times mg}{3 \times l\tan\theta}$$

Table 22. Estimates of labour required to erect stones

Stonehole	No. of people without levers	No of people with levers	No. of levers	Radius of levers (m)
F159	261	55	5	0.10
F283	197	40	4	0.10
F220	123	27	3	0.10
F227	91	20	2	0.10
F158	84	18	2	0.10
F146	76	16	2	0.09
F28	68	14	2	0.09
F215	65	14	2	0.09
F202	63	14	2	0.09
F17	61	14	2	0.09
F42	59	12	2	0.08
F66	57	12	1	0.11
F105	49	10	1	0.10
F230	42	9	1	0.10
F157	35	7	1	0.09
F139	34	7	1	0.09
F107	29	6	1	0.08
F154	27	6	1	0.08
F111	24	5	1	0.08
F138	24	5	1	0.08
F134	22	5	1	0.08
F160	22	5	1	0.08
F25	20	4	1	0.07
F19	19	4	1	0.07
F229	15	3	1	0.07
F207	15	3	1	0.07
F203	14	3	1	0.07
F226	13	3	1	0.07
F48	13	3	1	0.07
F219	12	3	1	0.07
Other Stones	3	1	1	0.05

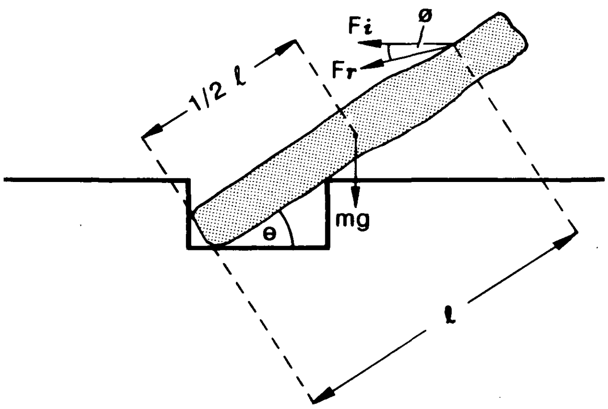

Figure 58 Taking moments about base of stone: erecting the stone

Correcting F_i we have

$$F_r = \frac{F_i}{\cos\phi}$$

or,

$$F_r = \frac{2 \times mg}{3 \times \tan\theta \cos\phi}$$

and the number of people required is I_m, where I_m is the smallest element of the set I defined by

$$I = \left\{ i \in N : i > \frac{F_r}{1000} \right\}$$

That is the smallest integer that is larger than the real number given by the formula.

To calculate the number of ropes required we use the formula

$$\sigma_r = \frac{F_r}{N_r \times A_r}$$

where σ_r is the tensile stress in each rope, N_r is the number of ropes and A_r is the cross sectional area of each rope. Equating this formula to the lower tensile strength of leather as given by Kaye and Laby (30×10^6 Nm^{-2}) and rearranging we have

$$N_r = \frac{F_r}{\sigma_r \times A_r}$$

To calculate the radius of a lever to produce a 5:1 increase in force we use the formula (Fig. 59)

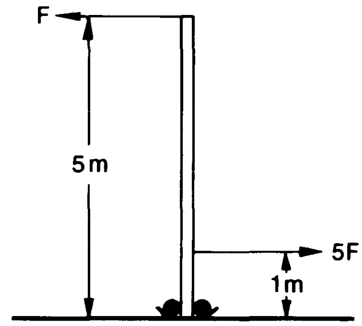

Figure 59 Force diagram to show lever used to effect 5:1 increase of force to bring stone upright

$$\sigma_{max} = \frac{4M}{\pi r^3} = \frac{16F}{\pi r^3}$$

where σ_{max} is the maximum stress in the lever, M is the maximum bending moment and F is the force exerted by the men. Equating this to the maximum stress for ash as given by Kaye and Laby (20×10^6 Nm^{-2}) and rearranging gives

$$r^3 = \frac{4F}{15\pi \times 10^6}$$

or,

$$r = \sqrt[3]{\frac{4F}{15\pi \times 10^6}}$$

To calculate number of people to lever up an end during extraction (Table 23)

Taking moments about A (Fig. 60) we have

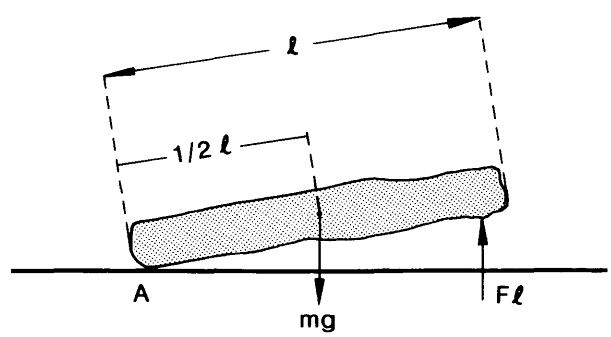

Figure 60 Taking moments about A: lifting the stone

$$F_l l = \frac{1}{2} l m g$$

where F_l is the force applied to the end of the stone. Rearranging the equation gives

$$F_l = \frac{mg}{2}$$

Table 23. Estimates of labour required to lift stones

Stonehole	No. of people with levers	No. of levers	Radius of lever (m)
F159	45	5	0.08
F283	36	4	0.07
F220	21	3	0.07
F227	16	2	0.07
F158	14	2	0.07
F146	14	2	0.07
F28	12	2	0.07
F215	12	2	0.07
F202	12	2	0.07
F17	10	1	0.08
F42	10	1	0.08
F66	10	1	0.08
F105	8	1	0.07
F230	7	1	0.07
F157	6	1	0.07
F139	6	1	0.07
F107	5	1	0.06
F154	5	1	0.06
F111	4	1	0.06
F138	4	1	0.06
F134	4	1	0.06
F160	4	1	0.06
F25	4	1	0.06
F19	4	1	0.06
F229	3	1	0.05
F207	3	1	0.05
F203	3	1	0.05
F226	3	1	0.05
F48	3	1	0.05
F219	2	1	0.05
Other Stones	1	1	0.03

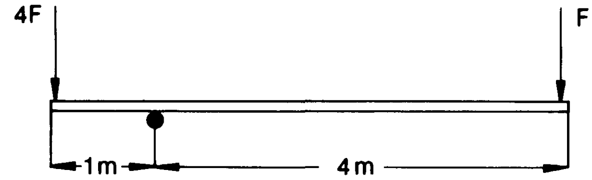

Figure 61 Force diagram to show 4:1 increase in force through use of a lever to lift one end of a stone

The number of people required to lift the stone with a 4:1 lever is I_m, where I_m is the smallest element of the set I defined by

$$I = \left\{ i \in N : i > \frac{F_l}{2000} \right\}$$

To calculate the radius of the levers to produce a 4:1 increase in force (Fig. 61) we again use the formula

$$\sigma_{\max} = \frac{4M}{\pi r^3} = \frac{16F}{\pi r^3}$$

and note that

$$F = \frac{F_l}{4N_l}$$

where N_l is the number of levers used. So equating to the to the maximum stress in wood and rearranging gives

$$r^3 = \frac{F_l}{15\pi N_l \times 10^6}$$

or,

$$r = \sqrt[3]{\frac{F_l}{15\pi N_l \times 10^6}}$$

To calculate the number of people required to move a stone on a sledge

Resolving forces and applying the properties of friction we have

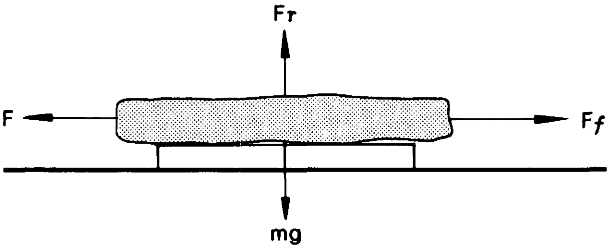

Figure 62 Force diagram for stone on sledge

$$F = F_f = \mu F_N = \mu mg$$

Worked Example with Stone 159

Length = 6.33 m, mass = 17469 kg, $g = 981\ \text{ms}^{-2}$, Θ = 24°, ϕ = 10°, $A_r = 5 \times 10^{-2}\ \text{m}^2$, μ = 201. Substituting into

$$F_r = \frac{2 \times mg}{3 \times \tan\theta \cos\phi}$$

gives

$$F_r = \frac{2 \times 17469 \times 9.81}{3 \times \tan 24 \cos 10}$$

or,

$$F_r = 260562 \text{ and } \frac{F_r}{1000} = 260.562$$

So the number of people required to erect the stone is 261. To calculate the number of ropes required we substitute into

$$N_r = \frac{F_r}{\sigma_r \times A_r}$$

so

$$N_r = \frac{260562}{30 \times 10^6 \times \pi \times 0.000625} = 4.42$$

that is five ropes are required to raise the stone.

We now calculate the radius of a lever to produce a 5:1 increase in force (Fig. 59) using

$$r = \sqrt[3]{\frac{4F}{15\pi \times 10^6}}$$

where F is the force exerted by 11 people on a lever, ie

$$r = \sqrt[3]{\frac{4 \times 11 \times 10^3}{15\pi \times 10^6}}$$

or,

$$r = 0.098$$

so five levers of radius 100 mm with 11 people per lever would be sufficient to raise this stone.

To lever up one end of the stone a force of F_l is applied, ie

$$F_l = \frac{17469 \times 9.81}{2}$$

or,

$$F_l = 85685$$

and

$$\frac{F_l}{2000} = \frac{85685}{2000} = 42.8$$

So at least 43 people are required to lift one end of the stone. To calculate the radius of the levers we use

$$r = \sqrt[3]{\frac{F_l}{15\pi N_l \times 10^6}}$$

if we suggest five levers with nine people per lever then

$$r = \sqrt[3]{\frac{85685}{15 \times 4 \times \pi \times 10^6}} = 0.077$$

So five levers of radius 80 mm with nine people per lever are sufficient to lift one end of the stone.

To move the stone on a sledge (Fig. 62) we use

$$F = \mu mg$$

ie $$F = 0.205 \times 17469 \times 9.81 = 35131.03$$

so a minimum of 36 people would be needed to move the stone.